XXIX. *Theory of the Parallaxes of Altitude for the Sphere, by* Mr. F. Mallet, *Profeſſor and Aſtronomer at* Upſal; *Tranſlated from the* French *by* M. Maty, *M. D. R. S. Sec.*

Upſal, October 25, 1766.

Read Nov. 20, 1766. § 1. LET P be = the moon's horizontal parallax, or 1 to ſin. P, as the moon's diſtance to the radius of the terreſtrial ſphere, on which the ſpectator is ſuppoſed to be placed. Let A be the diſtance of the moon from the zenith, and p the parallax of altitude for the ſame diſtance. The aſtronomers uſually compute the value of p in the following manner: let ſin. p=ſin. P. Sin. A, and p' being found by the tables of logarithmic ſines, ſin. p''=ſin. P. ſin. $\overline{A+p'}$ is found in like manner, p'' being aſſumed for the true parallax, which is not accurate.

§ 2. In order to ſhew this, I have given another method of computing the parallax of altitude, as exactly as may be, by means of the common tables, in the following manner. Since ſin. p = ſin. P ſin. $\overline{A+p}$, we have ſin. p = ſin. P ſin. A coſ. p + ſin. P coſ. A × ſin. p, or ſin. p (1 — ſin. P coſ. A) = ſin. P ſin. A coſ. p; hence tang. $p = \dfrac{\text{ſin. P ſin. A}}{1 - \text{ſin. P coſ. A}}$. This formula ſeems a little difficult to be wrought in numbers, but it is as eaſy as the above one; for, ſuppoſing ſin. B² = ſin.

fin. P cof. A, the tables will give the angle B, and tang. $p = \dfrac{\text{fin. P fin. A}}{\text{cof. B}^2}$, the computation of which can give no trouble. Hence it appears, that the calculus for finding the true parallax is not more difficult than that, which gives the faid parallax with an error, the value of which is unknown; for it is evident that the above computation for finding p'' is only an approximation, and that, to make it accurate, it would be neceſſary to carry it ſtill on by finding fin. $p''' =$ fin. P fin. $\overline{(A + p'')}$, and afterwards fin. $p'''' =$ P fin. $\overline{(A + p'')}$ &c.

§ 3. I therefore think myſelf in the right to prefer my method to that hitherto uſed by aſtronomers. To confirm my opinion, I made a trial, by putting P $= 59'$ and A $= 30°$, and found $p - p'' = 0'',43$, in which the error of the uſual computation amounts to near half a ſecond; I therefore give the preference to the geometrical calculus.

§ 4. Before I quit the formula tang. $p = \dfrac{\text{fin. P fin. A}}{1 - \text{fin. P cof. A}}$, I muſt obſerve, that the computation of p may be executed by other methods to the ſame exactneſs. If we take cof. 2 C $=$ fin. P cof. A, we ſhall have tang. $p = \dfrac{\text{fin. P fin. A}}{2 \, (\text{fin. C})^2}$, and the computation of this new formula is extremely eaſy.

§ 5. The formula tang. $p = \dfrac{\text{fin. P fin. A}}{1 - \text{fin. P cof. A}}$, gives befides, fin. $p = \dfrac{\text{fin. P fin. A}}{\sqrt{1 + \text{fin. P}^2 - 2\,\text{fin. P cof. A}}}$; make fin. P $= 2$ cof. D, D being a given angle, of which we may have tables ready made, and we ſhall have fin. $p =$

$$p = \frac{\text{fin. P fin. A}}{\sqrt{1 + 2 \text{ fin. P (cof. D--cof. A}}} = \frac{\text{fin. P fin. A}}{\sqrt{1 + 4 \text{ fin. P fin.} \frac{\text{A} + \text{D}}{2}}}$$

fin. $\frac{\text{A}-\text{D}}{2}$; fince cof. D--cof. A $= 2$ fin. $\frac{\text{A}+\text{D}}{2}$ fin. $\frac{\text{A}-\text{D}}{2}$.
This being found without any logarithmic compu-
tation, we fhall find tang. $\text{E}^2 = 4$ fin. P fin. $\frac{\text{A}+\text{D}}{2}$ fin.
$\frac{\text{A}-\text{D}}{2}$, if $\text{A} > \text{D}$, and hence we may eafily compute
fin. $p =$ fin. P fin. A cof. E; but if $\text{A} < \text{D}$ we fhall
find cof. $\text{F}^2 = 4$ fin. P fin. $\frac{\text{A}+\text{D}}{2}$ fin. $\frac{\text{A}-\text{D}}{2}$ and hence
fin. $p = \frac{\text{fin. P fin. A}}{\text{fin. F}}$.

§ 6. Similar formulæ may be found for cof. p, but
as the angle p is pretty fmall, one might eafily fall into
fome error by the ufual tables of logarithms. I fhall
not fay what would be the amount of this error of p,
having furnifhed the manner of avoiding it; but this
remark has not, I think, as yet been made in aftrono-
mical calculations ; and I have found it of great confe-
quence in computing eclipfes, where the diftances to be
found are very fmall arches.

§ 7. It may moreover be obferved, that if $\text{A} = \text{D}$,
fin. $p =$ fin. P fin. A ; hence $p' = p$ in the fame cafe,
and $p'' > p$, which feems very odd ; but the moon then
is below the fenfible horizon.

Theory of the apparent Diameters of the Moon.

§ 1. Firft the expreffion of horizontal diame-
ter of the moon, or of the diameter feen at the
horizon, feems to me too vague; for one ought to
underftand by it the diameter feen at the center of the
terreftrial

terreſtrial ſphere, rather than the apparent diameter at the horizon, which is not affected by refraction. Without this, if the one was confounded with the other, an error would ariſe for the latitude of Paris from o″,25 to o″,32.

§ 2. Let us keep the ſame denominations of P, p, and A, and call D the apparent ſemi-diameter of the moon at the centre of the ſphere, and $d =$ the apparent ſemi-diameter of the moon at the zenith diſtance $=$A. We ſhall have ſin. A : ſin. $\overline{A + p}$: : tang. D : tang. d, or if one will, ſin. A : ſin. $\overline{A + p}$: : D : d : the error not exceeding an 100th part of a ſecond.

§ 3. We had above ſin. $p =$ ſin. P ſin. $\overline{A + p}$. Hence ſin. P ſin. A : ſin. p : : (tang. D : tang. d) : : D : d, or becauſe ſin. $p = \dfrac{\text{coſ. } p \text{ ſin. P ſin. A}}{1 - \text{ſin. P coſ. A}}$, 1 — ſin. P coſ. A : coſ. p : : D, d, and $d = \dfrac{\text{D. coſ. } p}{1 - \text{ſin. P coſ. A}}$.

§ 4. Mr. Euler, in the Memoirs of the Academy of Berlin, 1747, pag. 175, makes this ſame value $= \dfrac{\text{V}}{1 - p^2 \text{ ſin. } b}$, and according to him, V $=$ D. M $=$ ſin. P ſin. $b =$ coſ. $\overline{A + p}$; from whence it appears, that the true value of the apparent diameter of the moon, is not more difficult to be computed than the approximated one of Mr. Euler, the exact and geometrical formula being tang. $d = \dfrac{\text{tang. D coſ. } p}{1 - \text{ſin. P coſ. A}}$ and that of Mr. Euler $d = \dfrac{\text{D}}{1 - \text{ſin. P coſ. } \overline{A + p}}$; for in both, the values of D, A and p muſt be employed.

§ 5.

§ 5. It likewife appears to me, that fince $\dfrac{\text{cof. } p}{1 - \text{fin. P cof. A}}$ $= \dfrac{\text{fin. } p}{\text{fin. P fin. A}}$ and therefore tang. $d = \dfrac{\text{tang. D fin. } p}{\text{fin. P fin. A}}$, aftronomers ought no lefs to employ this laft formula, than any other more troublefome, in practical computation. The fimpleft is tang. $d = \dfrac{\text{tang. D fin. } \overline{\text{A} + p}}{\text{fin. A}}$, upon the fuppofition of an exact table of the parallaxes of altitudes ready made; and I believe it will be as eafy to compute with tangents as with arches, by means of logarithms; and therefore this fimplification in putting arches inftead of tangents is unneceffary.

§ 6. To try the confequences of this theory, I made A$=$30°, D$=$1.5′, and taking the vertical of Upfal to the terreftrial axis for the radius of the fphere, I found P$=$55′,10″,3, fuppofing that the axis of the earth, is to the diameter of the equator as 199 to 200, and by the formulæ tang. $d = \dfrac{\text{tang. D fin. } \overline{\text{A} + p}}{\text{fin. A}} = \dfrac{\text{tang. D cof. } p}{1 - \text{fin. P cof. A}}$ $= \dfrac{\text{tang. D fin. } p}{\text{fin. P fin. A}}$, I found $d=$15′, 12″.664, but by the formula $d = \dfrac{\text{D cof. } p}{1 - \text{fin. P cof. A}}$, I had $d=$15′, 12″, 675. and laftly by that of Euler $d = \dfrac{\text{D}}{1 - \text{fin. P cof. } \overline{\text{A} + p}}$ we have $d=$15′,12″,635; from whence it appears that the error is very fmall, but that with the fame trouble one may avoid any error whatfoever.

§ 7. The prefent cafe did not give an error of 0″,001 in fubftituting 1 or the radius inftead of cof. p. Hence I conclude that $d = \dfrac{\text{D}}{1 - \text{fin. P cof. A}}$ will be

a more

a more exact formula than that of Euler $d=$
$$\frac{D}{1-\text{ſin. P coſ. } \overline{A+p}}.$$

§ 8. By taking $d=\dfrac{D}{1-\text{ſin. P coſ. A}}$, we have $d-D$
$=\dfrac{\text{D ſin. P coſ. A}}{1-\text{ſin. P coſ. A}}=\dfrac{\text{D ſin. } p \text{ coſ. A}}{\text{ſin. A coſ. } p}=\dfrac{\text{D tang. } p}{\text{tang. A}}$, which
affords an elegant theorem, to find the increaſe of the apparent diameter of the moon.

§ 9. I have found others by the following methods. Since ſin. A : ſin. $\overline{A+p}$ :: tang. D : tang. d, and ſin. A : ſin. $\overline{A+p}$ — ſin. A :: tang. D : tang. d — tang. D :: ſin. D coſ. d : ſin. $\overline{d-D}$; but coſ. D = coſ. d without any ſenſible error, and ſin. D coſ. D $=\frac{1}{2}$ ſin. 2 D, and ſin. $\overline{A+p}$ — ſin. A = 2 ſin. $\frac{1}{2}$ p coſ. $\overline{A+\frac{1}{2}p}$, we ſhall have ſin. $\overline{d-D}=\dfrac{\text{ſin. 2 D ſin. } \frac{1}{2}p \text{ coſ. } \overline{A+\frac{1}{2}p}}{\text{ſin. A}}$. In the ſame manner, as I before found ſin. $p'=$ ſin. P ſin. A and ſin. P ſin. A : ſin. p :: tang. D : tang. d, hence ſin. p' : ſin. p — ſin. p' :: ſin. p' : 2 ſin. $\overline{\frac{p-p'}{2}}$ coſ. $\overline{\frac{p+p'}{2}}$ :: $\frac{1}{2}$ ſin. 2 D : ſin. $\overline{d-D}=\dfrac{\text{ſin. 2 D ſin. } \overline{\frac{p-p'}{2}} \text{ coſ. } \overline{\frac{p+p'}{2}}}{\text{ſin. P ſin. A}}$.

§ 10. Laſtly let L = the diſtance of the moon from the center of the ſphere, l its radius, that of the ſphere being = 1, we have 1 : L :: ſin. P : 1 and L : l :: 1 : tang. D or 1 : l :: ſin. P : tang. D $= l$ ſin. P ; hence $l = \dfrac{\text{tang. D}}{\text{ſin. P}}$ being once found, ſince ſin. A : ſin. $\overline{A+p}$ :: tang. D : tang. d, and ſin. $\overline{A+p}$: ſin. p :: 1 : ſin. P, we ſhall have ſin. A : ſin. p :: tang. D : ſin. P tang. d :: l : tang. $d = \dfrac{l \text{ ſin. } p}{\text{ſin. A}}$. I found the logarithm of $l =$

9·4343965

9.4343965 at Upfal, by putting 10 for that of the radius of the fphere determined as before.

F. Mallet.

XXX. *A Catalogue of the* Fifty Plants *from* Chelfea Garden, *prefented to the* Royal Society *by the worfhipful Company of Apothecaries, for the Year* 1765, *purfuant to the Direction of* Sir Hans Sloane, *Bart. Med. Reg. et Soc. Reg. nuper Præfes : By* William Hudfon, *Societatis Regiæ & clariff. Societatis Pharmaceut.* Lond. *Soc. Hort.* Chelfean. *Præfectus et Prælector Botanic.*

2151 ACHYRANTHES lanata, caule proftrato, fpicis ovatis lateralibus, calycibus tomentofis. Lin. Sp. pl. 296. Mill. Dict. tab. 11. fig. 1.

Amaranthus Indicus verticillatus albus, foliis lanugine incanis. Pluk. alm. 27. tab. 79. f. 8.

2152 Andrachne procumbens herbacea. Lin. Sp. pl. 1439.

Telephoides Græcum humifufum flore albo. Tourn. cor. 50. Dill. Hort. Elth. 377. tab. 282 f. 364.

2153 Bryonia Africana, foliis palmatis quinquepartitis utrinque lævibus: laciniis pinnatifidis. Lin. Sp. pl. 1438.

Bryonia